# Motion in a Plane

1) State, for each of the following physical quantities, if it is a scalar or a vector: volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.

**Solution:**

Scalars: volume, mass, speed, density, number of moles, angular frequency.

Vectors: Acceleration, velocity, displacement, angular velocity.

2) Pick out the two scalar quantities in the following list:

Force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, relative velocity.

**Solution:**

Work and current are the two scalar quantities from the given list.

3) Pick out the only vector quantity in the following list:

Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.

**Solution:**

Impulse is the only vector quantity from the given list.

4) State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful:

(a) Adding any two scalars, (b) adding a scalar to a vector of the same dimensions,

(c) Multiplying any vector by any scalar, (d) multiplying any two scalars, (e) adding any two vectors, (f) adding a component of a vector to be the same vector.

**Solution:**

(a) No, only scalars of the same physical quantity can be added.

(b) No, a scalar cannot be added to a vector quantity, even if they have the same dimensions.

(c) Yes, linear momentum is the product of mass (scalar) and velocity (vector).

(d) Yes, the distance travelled by a body is the product of its speed and time, both being scalars.

(e) No, only vectors representing the same physical quantity can be added.

(f) Yes, only vectors representing the same physical quantity can be added.

5) Read each statement below carefully and state with reasons, if it is true or false:

(a) The magnitude of a vector is always a scalar, (b) each component of a vector is always a scalar, (c) the total path length is always equal to the magnitude of the displacement vector of a particle. (d) the average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time, (e) three vectors not lying in a plane can never add up to give a null vector.

**Solution:**

(a) True, the magnitude of a vector is a number with a unit, which is a scalar.

(b) False, a component of a vector is also a vector, as a component has a specific direction.

(c) False, it is true only when the particle moves along a straight line in the same direction.

(d) True, the total path length is always greater than or equal to the magnitude of a displacement vector. Hence, the average speed is greater than or equal to the magnitude of the average velocity over a given interval of time.

(e) True, they cannot be represented by a closed triangle.

6) A car moving along a straight highway with speed of 126 km $h^{-1}$ is brought to a stop within a distance of 200 m. what is the retardation of the car (assumed uniform), and how long does it take for the car to stop?

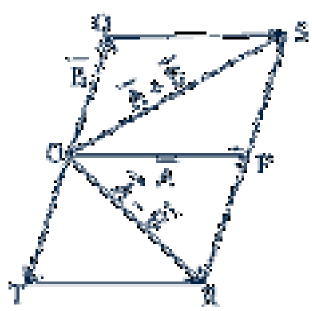
**Solution:**

Consider vectors $\vec{A}$ and $\vec{B}$ and let them be represented by sides $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ of a parallelogram OPSQ.

According to the parallelogram law of vector addition, $(\vec{A} + \vec{B})$ will be represented by $\overrightarrow{OS}$ as shown in the figure. Thus,

OP = $|\vec{A}|$, OQ = PS = $|\vec{B}|$

And OS = $|\vec{A} + \vec{B}|$



(a) To prove

$|\vec{A} + \vec{B}| \le |\vec{A}| + |\vec{B}|$

We know that the length of one side of a triangle is always less than the sum of the length of the other two sides. Hence, from ΔOPS, we have

OS < OP + PS (or) OS < OP+OQ (or) $|\vec{A} + \vec{B}| < |\vec{A}| + |\vec{B}|$ ----(1)

If the two vectors $\vec{A}$ and $\vec{B}$ are acting along the same straight line and in the same direction, then

$|\vec{A} + \vec{B}| = |\vec{A}| + |\vec{B}|$ -------(2)

Combining the conditions mentioned in (1) and (2),

We have $|a + b| < |a| + |b|$.

(b) To prove $|\vec{A} + \vec{B}| \geq |\vec{A}| - |\vec{B}|$

From ΔOPS, we have OS + PS > OP or OS > $|OP - PS|$ or OS >

$|OP - OQ|$ --------(3)

(PS = OQ)

The modulus of (OP – OS) has been taken because the LHS is always positive, but the RHS may be negative if OP < PS.

Thus from (3) we have

$|\vec{A} + \vec{B}| > |\vec{A}| - |\vec{B}|$ ------(4)

If the two vectors $\vec{A}$ and $\vec{B}$ are acting along a straight line in opposite directions, then,

$|\vec{A} + \vec{B}| = |\vec{A}| - |\vec{B}|$ ---------(5)

Combining the conditions mentioned in (4) and (5), we get

$|\vec{A} + \vec{B}| \geq |\vec{A}| - |\vec{B}|$

(c) To prove $|\vec{A} - \vec{B}| \leq |\vec{A}| - |\vec{B}|$

From figure, $\vec{A} = \overrightarrow{(OP)}$ and $\overrightarrow{-B} = \overrightarrow{OT} = \overrightarrow{PR}$ and $\overrightarrow{(A - B)} = \overrightarrow{OR}$

From ΔORP, we note that OR < OP + PR

Or $|\vec{A} - \vec{B}| < |\vec{A}| + |-\vec{B}|$ or $|\vec{A} - \vec{B}| < |\vec{A}| + |\vec{B}|$ --------(6)

If the two vectors are acting along a straight line, but in the opposite directions, then

$|\vec{A} - \vec{B}| < |\vec{A}| + |\vec{B}|$ --------(7)

Combining the conditions mentioned in (6) and (7), we get

$|\vec{A} - \vec{B}| \leq |\vec{A}| + |\vec{B}|$

(d) To prove $|\vec{A} - \vec{B}| \geq |\vec{A}| - |\vec{B}|$

From figure, in ΔOPR, we note that

OR + PR > OP or OR > $|OP - PR|$ or OR > $|OP - OT|$ -------(8)

The modulus of (OP – OT) has been taken because the LHS is positive and the RHS may be negative if OP < OT.

From (8), $|\vec{A} - \vec{B}| > |\vec{A}| - |\vec{B}|$ --------(9)

If the two vector are acting along the same straight line in the same direction, then

$|\vec{A} - \vec{B}| = |\vec{A}| - |\vec{B}|$ --------(9)

Combining the conditions mentioned in (9) and (10), we get

$|\vec{A} - \vec{B}| \geq |\vec{A}| - |\vec{B}|$ ----------(9)

7) Given a + b + c + d=0, which of the following statements are correct:

(a) a, b, c and d must each be a null vector,

(b) The magnitude of (a + c) equals the magnitude of (b + d)

(c) The magnitude of a can never be greater the sum of the magnitude of b, c, and d

(d) b + c must lie in the plane of a and d if a and d are not collinear, and in the line of a and d, if they are collinear?

**Solution:**

(a) Not correct, $\bar{a} + \bar{b} + \bar{c} + \bar{d}$ can be a null vector in many ways.

Each vector need not be a null vector, but the sum of all the vectors can be zero, as the direction of any one of the vectors can be opposite to the resultant of the other vectors.

(b) Correct $\bar{a} + \bar{b} + \bar{c} + \bar{d} = 0$

$= (\bar{a} + \bar{c}) = - (\bar{b} + \bar{d})$

$= | \bar{a} + \bar{c} | = - | \bar{b} + \bar{d} |$

(c) Correct $\bar{a} + \bar{b} + \bar{c} + \bar{d} = 0$

$\bar{a} = - (\bar{b} + \bar{c} + \bar{d})$

$\bar{a} = (\bar{b} + \bar{c} + \bar{d})$

This means that the magnitude of $\bar{a}$ is equal to the magnitude of the vector

$\bar{b} + \bar{c} + \bar{d}$ the magnitude of the vector $\bar{b} + \bar{c} + \bar{d}$ is less than or equal to the sum of the magnitudes of $\bar{b}$, $\bar{c}$ and $\bar{d}$ is less than or equal to the sum of the magnitudes of $\bar{b}$, $\bar{c}$ and $\bar{d}$

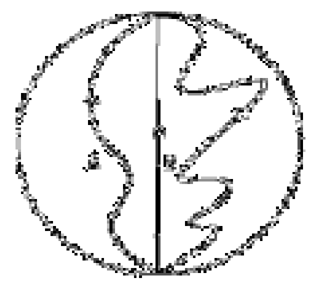
(d) Correct $\bar{a}$, $(\bar{b} + \bar{c})$ and $\bar{d}$ are three vectors.

(1) The sum of $\bar{a}$ , ($\bar{b}$ + $\bar{c}$ ) and $\bar{d}$ will be a null vector, if they are coplanar.

(2) If $\bar{a}$ , $\bar{d}$ are collinear, then the resultant of $\bar{a}$ , ($\bar{b}$ + $\bar{c}$ ) and $\bar{d}$

Will be a null vector, if $\bar{b}$ + $\bar{c}$ is on the line of $\bar{a}$ and $\bar{d}$ .

8) Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in fig. 4.29. What is the magnitude of the displacement vector for each? For which girl is this equal to the actual length of path skate?



**Solution:**

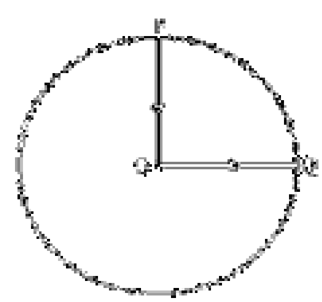
As the initial and final positions are same for all the three girls, their displacement is the same, which is equal to length of the diameter $\underset{(PQ)}{\longrightarrow}$ magnitude of the displacement = 2r

=2 x 200

=400 m

For girl ‘B’, the actual length of the path skated and the magnitude of displacement are equal.

9) A cyclist starts from the centre O of a circular park of radius 1km, reaches the edge P of the park, then cycles along the circumference, and returns to the centre along QO as shown in fig. 4.21, if the round trip take 10 min, what is the (a) net displacement, (b) average velocity, and (c) average speed of the cyclist?



**Solution:**

Radius of the circular path, r = 1 km = 1000 m

Time taken to complete the trip, t =10 minute = 600 s

(a) As the cyclist returns to the initial point, his net displacement is zero. (null vector)

(b) Average velocity = $\frac{net\ displacement}{total\ time\ taken}$

= 0 (net displacement = 0)

(c) Average speed of the cyclist $\frac{total\ distance}{total\ time\ taken}$

Total distance travelled = OP + arc length PQ + OQ

$=2r + \frac{\pi r}{2}$

$=2000 + 500\pi$

Total distance travelled =2000 + 1570

3570 m

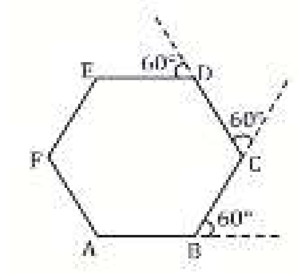
Average speed = $\frac{3570}{600}$ m $s^{-1}$

=5.95 m $s^{-1}$

10) On an open ground, a motorist follows a track that turns to his left by an angle of 600 after every 500 m. starting from a given turn; specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.

**Solution:**

Let us consider a motorist staring from point 'A'. He covers 500 m and takes the first turn at B, the second turn at 'C' and the third turn at 'D'. On successive turns at the end of every 500 m, his path is a closed regular hexagon ABCDEFS as shown in the figure.




(1) At the third turn, the distance travelled is 3 x 500 = 1500 m

Displacement, $\underset{AB}{\rightarrow} + \underset{BC}{\rightarrow} + \underset{CD}{\rightarrow} = \underset{AD}{\rightarrow}$

This is the diagonal of the regular hexagon.

The length of the diagonal is twice its side, hence AD= 1000 m. diagonal AD is parallel to side BC. Hence, the displacement at the third turn is at $60^0$ from the initial direction. The total length is 1.5 times the displacement.

(2) If the motorist starts from A and takes six turns successively at B, C, D, E, F, A. he moves along a regular hexagon of side 500 m. as his initial and final position is at 'A' at the sixth turn, his displacement is a null vector.

Distance travelled by him at the sixth turn = 6 x 500 = 3000 m

(3) At the $8^{th}$ turn, he will be at point 'C'.

His displacement is, $\underset{AC}{\rightarrow} = \underset{AB}{\rightarrow} + \underset{BC}{\rightarrow}$

$=\sqrt{3}$ x 500

(Using geometry of triangles)

=866 m

Distance travelled = 8 x 500

=4000 m

11) A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 min. what is (a) the average speed of the taxi, (b) the magnitude of average velocity? Are the two equal?

**Solution:**

Let us consider 'A' as the location of the station and 'B' as the location of the hotel. The cabman takes the passenger along the path ACDB.

(a) Average speed = $\frac{total\ distance}{total\ time\ taken}$

$=\frac{23{,}000}{28 \times 60}$

$=13.69\ m\ s^{-1}$

(b) Average velocity = $\frac{net\ displacement}{total\ time\ taken}$

$=\frac{10{,}000}{28 \times 60}$

$=5.95\ m\ s^{-1}$

The average speed and the average velocity are not equal. As the distance is greater than the displacement, the average speed is greater than the average velocity.

12) Rain is falling vertically with a speed of 30 m $s^{-1}$. A woman rides a bicycle with a speed of 10 m $s^{-1}$ in the north to south direction. What is the direction in which she should hold her umbrella?

**Solution:**

Speed of the rain, $v_R = 30\ m\ s^{-1}$

Speed of the woman, $v_W$= 10 m $s^{-1}$

The woman has to hold the umbrella in the direction of resultant velocity of rain with respect to herself.

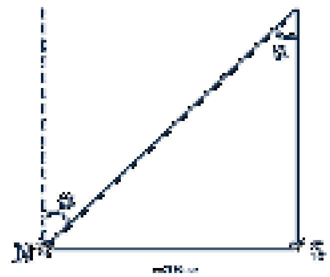
If ‘α’ is the angle between the direction of the resultant velocity of the rain with the vertical, then

Tan α = $\frac{v_W}{v_R}$

=$\frac{10}{30}$

α = $\tan^{-1}$ ($\frac{1}{3}$)

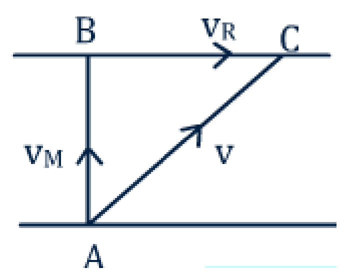
α = $18.43^0$



Thus, the woman should hold the umbrella at an angle of $18.43^0$ with the vertical towards the south.

13) A man can swim with a speed of 4.0 km/h in still water. How long does he take to cross a river 1.0 km wide if the river flows steadily at 3.0 km/h and he makes his strokes normal to the river current? How far down the river does he go when he reaches the other bank?

**Solution:**




According to the problem,

Speed of the man, $v_M$= 4.0 kmph

Speed of the river water, $v_R$ = 3.0 kmph

Width of the river, d= 1 km

The speed of the man with respect to the bank of the river, v= $\sqrt{v_M^2 + v_R^2}$

Time taken to cross the river, t= $\frac{AB}{V_M}$

=$\frac{1}{4}$ hour

t = 0.25 hour

As the man strokes normal to the river, he is carried down the river (drift).

Drift of the man, BC = $v_R$ x t

=3 x (0.25)

=0.75 km

Thus, the man takes 0.25 hour to cross the river and in this time he is carried 0.75 km down the river.

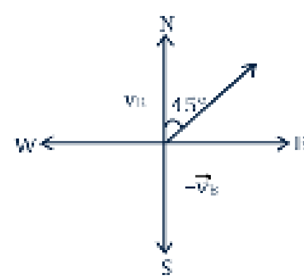
14) In a harbor, wind is blowing at the speed of 72 km/h and the flag on the mast of a boat anchored in the harbor flutters along the N-E direction. If the boat starts moving at a speed of 51 km/h to the north, what is the direction of the flag on the mast of the boat?

**Solution:**

According to the problem, speed of the wind, $v_W$ =72 kmph

As the flag on the anchored boat flutters along the N-E direction, the direction of wind is N-E.

Speed of the boat, $v_B$ = 51 km along the north. The flag on the mast of the boat is in the direction of the wind with respect to the boat.




$v_{WB} = \underset{V_W}{\longrightarrow} - \underset{V_B}{\rightarrow}$

$= \underset{V_W}{\longrightarrow} + \underset{V_B}{\rightarrow}$

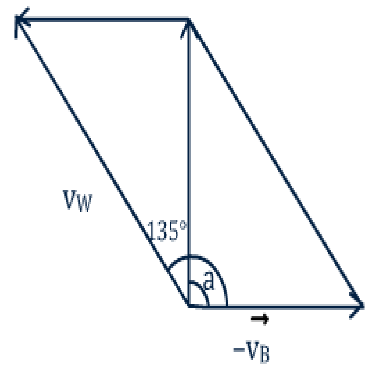
Let 'α' be the angle made by $\underset{V_W}{\longrightarrow}$ with $- \underset{V_B}{\rightarrow}$

Tan α = $\dfrac{\left|\overrightarrow{v_W} \sin 135^0\right|}{\left|\overrightarrow{-v_W}\right| + \left|\overrightarrow{v_W}\right| \cos 135^0}$

Tan α = $\dfrac{72 \times \left(\frac{1}{\sqrt{2}}\right)}{51 + 72\left(-\frac{1}{\sqrt{2}}\right)}$

$= \dfrac{51}{0.08}$

$\alpha = 90^0$ (approximately)




Hence, the flag flutters approximately in the direction of east.

15) The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 m s$^{-1}$ can go without hitting the ceiling of the hall?

**Solution:**

According to the problem,

Height of the ceiling, h=25 m

Speed of the ball, u= 40 m s$^{-1}$

The maximum height the ball can reach, H= h = 25 m

Let 'θ' be the angle of projection of the ball, with the horizontal so that it reaches the required height.

$\dfrac{u^2 \sin^2 \theta}{2g} = 25$

$\sin^2 \theta = \dfrac{25 \times 2 \times 9.8}{40 \times 40}$

$\sin^2 \theta = \dfrac{49}{160}$

sin θ = 0.5534

$\theta = 36.5^0$

As $\theta < 450$, the horizontal range of the ball, R = $\frac{u^2 sin^2 \theta}{2g} = 25$

$= \frac{(40)^2 \sin 67.2}{9.8}$

R= 150.51 m

16) A cricketer can throw a ball to a maximum horizontal distance of 100 m. how much high above the ground can the cricketer throw the same ball?

**Solution:**

Maximum horizontal distance, $R_{max} = \frac{u^2}{g} = 100$ m

The maximum height reached by the ball is, H= $\frac{u^2}{g}$

H= $\frac{u^2}{g}$

H= 50 m

17) A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone?

**Solution:**

According to the problem,

Radius of the horizontal circle, r = 80 cm

r = 0.8 m

Number of revolution per second, v = $\frac{14}{15}$ rps

As the stone is revolving with uniform speed, the acceleration produced in it is due to the change in direction. This acceleration is called centripetal acceleration or radial acceleration, which is towards the centre of the circular path.

$a_r = \frac{v^2}{r}$ of r $\omega^2$

$a_r = (0.8)\left(2\pi \times \frac{14}{25}\right)^2$ (since, $\omega = 2\pi v$)

$a_r = 9.9$ m $s^{-2}$

Thus, the acceleration of the stone is 9.9 m $s^{-2}$ towards the centre of the circular path.

18) A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone?

**Solution:**

According to the problem,

Radius of the loop, r = 1 km = 1000 m

Speed of the air craft, v = 900 kmph

v = 900 x $\frac{5}{18}$ m $s^{-1}$

v = 250 m $s^{-1}$

**Centripetal acceleration, $a_r = \frac{v^2}{r}$**

$= \frac{250 \times 256}{1000}$

$a_r = 62.5$ m $s^{-2}$

Acceleration due to gravity is 9.8 m $s^{-2}$

Ratio of '$a_r$' to 'g' = $\frac{62.5}{9.8}$ = 6.4

Hence, centripetal acceleration is 6.4 times acceleration due to gravity.

19) Read each statement below carefully and state, with reasons, if it is true or false:

(a) The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre.

(b) The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point.

(c) The acceleration vector of a particle is uniform circular motion averaged over one cycle is a null vector.

**Solution:**

(a) False. For a particle in circular motion, the acceleration is towards the centre of the circle only when it moves with uniform speed.

(b) True. The net displacement over one cycle is a null vector. Hence, acceleration is also a null vector.

20) The position of a particle is given by

$r = 3.0t\,\hat{\imath} - 2.0t^2\,\hat{\jmath} + 4.0\,\hat{k}$ m

Where t is in seconds and the coefficients have the proper units for r to be in metres.

(a) Find the v and a of the particle?

(b) What is the magnitude and direction of velocity of the particle at t = 2.0 s?

**Solution:**

The position vector of the particle, $\underset{r}{\rightarrow}$ = 3.0t $\hat{\imath}$ - 2.0t$^2$ $\hat{\jmath}$ + 4.0 $\hat{k}$

(a) Velocity vector of the particle $\underset{v}{\rightarrow} = \frac{\overrightarrow{dr}}{dt}$

$\underset{v}{\rightarrow}$ = 3.0 $\hat{\imath}$ - 4.0t $\hat{\jmath}$ m s$^{-1}$

Acceleration of the particle, $\underset{a}{\rightarrow} = \frac{\overrightarrow{dv}}{dt}$

$\underset{a}{\rightarrow}$ = -40 m s$^{-2}$

(b) Velocity vector of the particle, at t = 2 s is

$\underset{v_{t=2}}{\longrightarrow}$ = 3.0 $\hat{\imath}$ - 8.0 $\hat{\jmath}$ m s$^{-1}$

$\underset{|v|}{\longrightarrow} = \sqrt{9+64}$

=$\sqrt{73}$ m s$^{-1}$

= 8.54 m s$^{-1}$

Tan$\theta = \frac{-8}{3}$

$\theta$ = tan-1 $\left(\frac{-8}{3}\right)$

Thus, at t=2 s, the velocity of the particle is 8.53 m s$^{-1}$ and it makes an angle of -69.4$^0$ with the positive x-axis or 20.6$^0$ with the negative y-axis.

21) A particle starts from the origin at t=0 s with a velocity of 10.0 $\hat{j}$ m/s, and moves in the x-y plane with a constant acceleration of $(8.0\hat{i} + 2.0\hat{j})$ m $s^{-2}$.

(a) At what time is the x-coordinate of the particle 16 m? What is the y-coordinate of the particle at that time?

(b) What is the speed of the particle at the time?

**Solution:**

According to the problem,

Velocity of the particle, at t=0 s, $\underset{u}{\rightarrow} = 100\hat{j}$ m $s^{-1}$

Acceleration of the particle, $\underset{a}{\rightarrow} = 8.0\hat{i} + 2.0\hat{j}$ m $s^{-2}$

(a) x =16$\hat{i}$ when t = ?

From the equation, $s = ut + \frac{1}{2}at^2$

$16\hat{i} = (10.0\,\hat{j})\,t + \frac{1}{2}(8.0\hat{i} + 2.0\hat{j})\,t^2$

Equating the $\hat{i}$ components on both sides of the equation, we get

$16i = 4.0\,t^2\,\hat{i}$

$t^2 = 4$

$t = 2$

The position of the particle at t=2 s is given by

$\hat{s} = (10.0\,\hat{i})\,2 + \frac{1}{2}(8.0\hat{i} + 2.0\hat{j})\,4$

$\hat{s} = 16\hat{i} + 24\hat{j}$

Hence, when t=2 s, the x-coordinate of the particle is 16 m and the y-coordinate of the particle is 24 m.

(b) Speed of the particle at x=16$\hat{\imath}$ , t = 2 s

From the equation, $\vec{v} = \vec{u} + \vec{at}$ , we get

$\vec{v}$ = 10.0 $\hat{\jmath}$ + (8.0$\hat{\imath}$ + 2.0$\hat{\jmath}$) (2)

$\vec{v}$ = 16.0 $\hat{\imath}$ + 14.0 $\hat{\jmath}$

Sped of the particle = $\overrightarrow{|v|}$

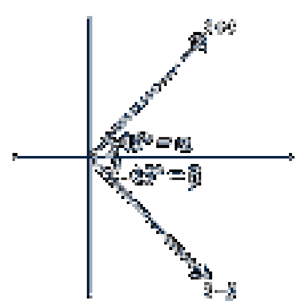
= $\sqrt{(16)^2 + (14)^2}$

=$\sqrt{256 + 196}$

$\overrightarrow{|v|}$ = 21.26 m $s^{-1}$

22) $\hat{\imath}$ and $\hat{\jmath}$ are unit vectors along x- and y-axis respectively. What is the magnitude and direction of the vectors $\hat{\imath}$ + $\hat{\jmath}$, and $\hat{\imath}$ - $\hat{\jmath}$ ? What are the components of a vector A= 2 $\hat{\imath}$ + 3 $\hat{\jmath}$ along the directions of $\hat{\imath}$ + $\hat{\jmath}$ and $\hat{\imath}$ - $\hat{\jmath}$ ? (You may use graphical method)

**Solution:**



(1) Magnitude of the vector $\hat{\imath}$ + $\hat{\jmath}$ = $\sqrt{2}$

Let the angle made by $\hat{\imath}$ + $\hat{\jmath}$ with the positive x-axis be ‘α’.

Tan α = $\frac{1}{1}$

α= $45^0$

(2) Magnitude of the vector $\hat{\imath}$ - $\hat{\jmath}$ = $\sqrt{(1)^2 + (-1)^2}$ = $\sqrt{2}$

Let the angle made by $\hat{\imath}$ - $\hat{\jmath}$ with the positive x-axis be ‘β’.

Tan β = $\frac{-1}{1}$

β = $45^0$

(3) Let 'a' be the unit vector along $\hat{\imath} + \hat{\jmath}$ then $\hat{a} = \frac{\hat{\imath} + \hat{\jmath}}{\sqrt{2}}$

The component of $2\hat{\imath} + 3\hat{\jmath}$ along $\hat{\imath} + \hat{\jmath}$ = $(2\hat{\imath} + 3\hat{\jmath}).\ \hat{a}$

$(2\hat{\imath} + 3\hat{\jmath}).\ (\frac{\hat{\imath} + \hat{\jmath}}{\sqrt{2}})$

$= \frac{1}{\sqrt{2}} (2\hat{\imath} + 3\hat{\jmath}).\ (\hat{\imath} + \hat{\jmath})$

$= \frac{2+3}{\sqrt{2}}$

$= \frac{5}{\sqrt{2}}$

Let $\hat{b}$ be the unit vector along $\hat{\imath}$ - $\hat{\jmath}$ then $\hat{b} = \frac{\hat{\imath} - \hat{\jmath}}{\sqrt{2}}$

The component of $2\hat{\imath} + 3\hat{\jmath}$ along $\hat{\imath}$ - $\hat{\jmath}$ = $(2\hat{\imath} + 3\hat{\jmath}).\ \hat{b}$

$=(2\hat{\imath} + 3\hat{\jmath}).\ (\frac{\hat{\imath} + \hat{\jmath}}{\sqrt{2}})$

$= \frac{1}{\sqrt{2}} = (2\hat{\imath} + 3\hat{\jmath}).\ \hat{\imath} - \hat{\jmath}$

$= \frac{2-3}{\sqrt{2}}$

$= -\ \frac{1}{\sqrt{2}}$

23) For any arbitrary motion in space, which of the following relations are:

(a) $v_{average} = (1/2)\ (v(t_1) + v\ (t_2))$

(b) $v_{average} = [r(t_2) - r(t_1)]/(t_2 - t_1)$

(c) $v\ (t) = v(0) + at$

(d) $r(t) = r(0) + v(0)\ t + (1/2)\ at_2$

(e) $a_{average} = [v(t_2) - v(t_1)]/(t_2 - t_1)$

(The 'average' stands for average of the quantity over the time interval $t_1$ to $t_2$)

**Solution:**

Only (b) and (e) are true for any arbitrary motion in space.

(a), (c) and (d) are true only for uniformly accelerated motions.

24) Read each statement below carefully and state, with reasons and examples, if it is true or false:

A scalar quantity is one that

(a) is observed in a process

(b) can never take negative values

(c) must be dimensionless

(d) does not vary from one point to another in space

(e) has the same value for observers with different orientations of axes.

**Solution:**

(a) False. Kinetic energy is a scalar, but it is not conserved in an inelastic collision.

(b) False. Temperature is a scalar, and can take negative values. The negative value for a scalar does not indicate an opposite direction, but indicates a value less than zero.

(c) False. Volume is scalar, but it has dimensions.

(d) False. The potential due to elastic, magnetic and gravitational field is scalar, but it varies from point to point in the corresponding fields.

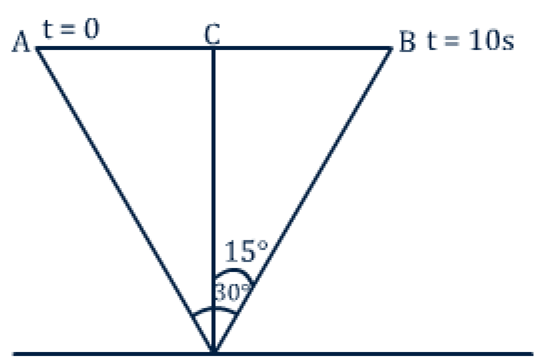
(e) True. The value of a scalar does not change with orientation of axes. For example, the speed of light is same in all directions.

25) An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10.0 s apart is $30^0$, what is the speed of the aircraft?

**Solution**:

Let us consider that the aircraft is at 'A' when t=0 s and 'B' when t=10 s. according to the problem,

OC= 3400 m




Angle AOB = $30^0$

From the right angle OCA

Tan $15^0$ = AC/OC

AC = (OC) Tan $15^0$

AC =3400 x tan $15^0$

AC = 911 m

If 'v' is the speed of air craft, then

AB =v x 10

2AC =v x 10

$2 \times 911 = 10 \, v$

$v = 182.2 \text{ m s}^{-1}$

26) A vector has magnitude and direction. Does it have a location in space? Can it vary with time? Will two equal vectors a and b at different locations in space necessarily have identical physical effects? Give examples in support of your answer.

**Solution:**

(1) No. in general, a vector does not have a definite location in space. However, a position vector has a definite location in space.

(2) Yes. The velocity and momentum vectors of an accelerated particle vary with time.

(3) No. for example, when two equal forces are applied at different locations on a body, the turning effect produced by them is not necessarily the same.

27) A vector has both magnitude and direction. Does it mean that anything that has magnitude and direction is necessarily a vector? The rotation of a body can be specified by the direction of the axis of rotation, and the angle of rotation about the axis. Does that make any rotation a vector?

**Solution:**

(1) No. a physical quantity having both magnitude and direction is called a vector only if it obeys the laws of vector addition.

For example, electric current, stress.

(2) The rotation of a body is not a vector, but for an infinitely small rotation, angular displacement is a vector.

28) Can you associate vectors with (a) the length of a wire bent into a loop, (b) a plane area, (c) a sphere? Explain.

**Solution:**

(a) No, we cannot associate a vector with the length of a wire bent into a loop.

(b) Yes, the area vector of a plane has direction normally outwards to the plane.

(c) We cannot associate a vector to the volume of a sphere, but a vector can be associated with the surface area of a sphere.

29) A bullet fired at an angle of 300 with the horizontal hits the ground 3.0 km away. By adjusting its angle of projection, can one hope to hit a target 5.0 km away? Assume the muzzle speed to the fixed, and neglect air resistance.

**Solution:**

According to the problem,

Angle of projection, $\theta=30^0$

Horizontal range, R= 3.0 km

$R= u^2 \sin 2\theta/g$

$3.0 = (u^2/g) \times \sin 60^0$

$\frac{3\times2}{\sqrt{3}} = 2\sqrt{3}$

$u^2/g = 3.46$ km

$R_{max} = u^2/g = 3.46$ km

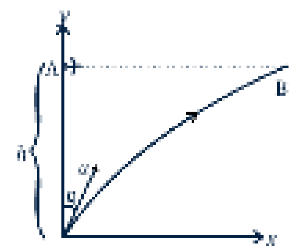
As $R_{max} < 5.0$ km we cannot hit a target 5.0 km away with the same muzzle speed, even by altering the angle of projection.

30) A fighter plane flying horizontally at an altitude of 1.5 km with speed 720 km/h passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed 600 m $s^{-1}$ to hit the plane? At what minimum altitude should the pilot fly the plane to avoid being hit? (Take g = 10 m $s^{-2}$).

**Solution:**

According to the problem,

Altitude of the plane, h= 1.5 km = 1500 m



Speed of the plane, v= 720 kmph

v = 720 x $\frac{5}{18}$ m $s^{-1}$

v = 200 m $s^{-1}$

Muzzle speed of the shell fired from the gun, u= 600 m $s^{-1}$

Consider that the shell will hit the plane at 'B' at the end't' seconds, if it is fired at an angle '$\theta$' with the vertical when the aircraft is exactly above the gun. Then, for the shell to hit the target, the horizontal displacement of the shell should be equal to the displacement of the aircraft in the time interval during which the shell gains the altitude of the aircraft.

(1) Horizontal distance travelled by shell = distance travelled by plane (u sin $\theta$)t=vt

sin $\theta$ = v/u

sin $\theta$ = $\frac{200}{600} = \frac{1}{3}$

$\theta$ = $19.47^0$

(2) The plane will not be hit by the shell, if h≥ $H$, where 'h' is the maximum height gained by the shell.

The maximum height gained by the shell H= $\frac{u^2 cos^2 \theta}{2g}$

$H = \frac{(600)^2 \times cos^2(19.47^0)}{2 \times 10}$

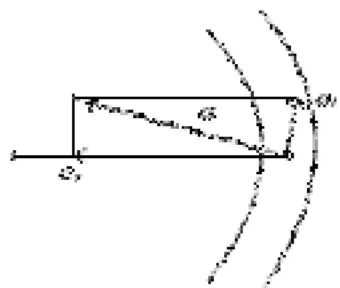
=16000 m

H= 16 km

Thus, the minimum altitude at which the pilot should fly the plane to avoid being hit by a shell is 16 km.

31) A cyclist is riding with a speed of 27 km/h. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate of 0.50 m/s every second. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn?

**Solution:**



According to the problem,

Speed of the cyclist, v=27 kmph = 7.5 m $s^{-1}$

Radius of the circular path, r = 80 m

The rate at which speed is reduced = 0.5 m $s^{-2}$

The direction in which the speed decreases is along the tangent to the circle. So, the acceleration due to change in speed is called tangential acceleration. Tangential acceleration, at = 0.5 m $s^{-2}$

The direction in which the speed decreases is along the tangent to the circle. So, the acceleration due to change in speed is called tangential acceleration. Tangential acceleration, at=0.5 m $s^{-2}$

Radial acceleration of the cyclist, $a_r = \frac{v^2}{r}$

$a_r = \frac{7.5 \times 7.5}{80}$

= 0.7 m $s^{-2}$

Net acceleration of the cyclist, a= $\sqrt{a_r^2 + a_t^2}$

a = $\sqrt{(0.7)^2 + (0.5)^2}$

a = 0.86 m $s^{-2}$

Let us consider that the net acceleration makes an angle '$\theta$' with $\overline{a_r}$,

Then, tan $\theta = \frac{a_1}{a_r}$

Tan $\theta = \frac{0.5}{0.7}$

$\theta$ = $\text{Tan}^{-1}$ $(\frac{5}{7})$

$\theta = 35.54^0$

Hence, the acceleration of the cyclist on the circular turn is 0.86 m $s^{-2}$, and it makes an angle of $35.54^0$ with the radius vector.

32) (a) Show that for a projectile the angle between the velocity and the x-axis as a function of time is given by

$\theta$ (t) = $\tan^{-1}$ $(\frac{v_{oy} - gt}{v_{ox}})$

(b) Shows that the projectile angle $\theta_0$ for a projectile launched from the origin is given by

$\theta_0 = \tan^{-1}(\frac{4h_m}{R})$

Where the symbols have their usual meaning.

**Solution:**

Let us consider a projectile launched with velocity '$v_0$' that makes an angle '$\theta_0$' with the x-axis. If we take the point of projection as the origin of the reference frame, then

$x_0 = 0$ and $y_0 = 0$

the components of initial velocity $v_0$ are:

(1) along the x-axis, $v_{0x} = v_o \cos\theta_0$

(2) along the y-axis, $v_{oy} = v_o \sin\theta_0$

The components of velocity at the end of 't' seconds are:

(1) along the x-axis $v_x = v_o \cos\theta_0$ (since acceleration along the x-axis is zero)

(2) along the y-axis, $v_y = v_o \sin\theta_0 - gt$

The angle between the velocity and the x-axis as a function of time is given by

$$\theta_{(t)} = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

$$\theta_{(t)} = \tan^{-1}\left(\frac{v_{0y} - gt}{v_{0x}}\right)$$

The maximum height '$h_m$' reached by the projectile, $h_m = \frac{(v_0 \sin\theta_0)2}{2g}$

The horizontal distance travelled by the projectile, $R = \frac{2v_0^2 \sin\theta_0 \cos\theta_0}{g}$

$$\frac{h_m}{R} = \frac{\frac{(v_0 \sin\theta_0)2}{2g}}{\frac{2v_0^2 \sin\theta_0 \cos\theta_0}{g}}$$

$$\frac{h_m}{R} = \frac{\tan\theta_0}{4}$$

$$\theta_0 = \tan^{-1}\left(\frac{4h_m}{R}\right)$$